

In eq.s (A.13), (A.14), (A.15) the term $-3zH$ should be $-3H$.

In eq.s (A.18) and (A.19) the quantity $\widehat{\Gamma}_i$ should be $\widetilde{\Gamma}_i \equiv d\Gamma_i/dz$.

Equation (A.17) should be (note, in the denominator, $\hat{\rho}_{e\gamma}$ instead of $\hat{\rho}_{e\gamma B}$)

$$\frac{d\phi_e}{dz} = \frac{1}{z} \frac{\hat{L} \kappa_1 + \left(\hat{\rho}_{e\gamma B} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right) \kappa_2}{\hat{L} \frac{\partial \hat{\rho}_e}{\partial \phi_e} - \frac{\partial \hat{L}}{\partial \phi_e} \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right)} \quad (1)$$

where

$$\begin{aligned} \kappa_1 &= 4(\hat{\rho}_e + \hat{\rho}_\gamma) + \frac{3}{2} \hat{p}_B - z \frac{\partial \hat{\rho}_e}{\partial z} - z \frac{\partial \hat{\rho}_\gamma}{\partial z} + \frac{1}{\hat{L}} \left(3\hat{L} - z \frac{\partial \hat{L}}{\partial z} \right) \hat{\rho}_B - \frac{z^2 \hat{L}}{\sum_j Z_j X_j} \sum_i \left(\Delta \widehat{M}_i + \frac{3}{2z} \right) \widetilde{\Gamma}_i \\ \kappa_2 &= z \frac{\partial \hat{L}}{\partial z} - 3\hat{L} - z \hat{L} \frac{\sum_i Z_i \widetilde{\Gamma}_i}{\sum_j Z_j X_j} \end{aligned} \quad (2)$$

The same is true for equation (A.22), where the identical denominator appears

$$\frac{dX_i}{dz} = \dot{X}_i \frac{dt}{dz} = - \frac{\widehat{\Gamma}_i}{3z \widehat{H}} \frac{\kappa_1 \frac{\partial \hat{L}}{\partial \phi_e} + \kappa_2 \frac{\partial \hat{\rho}_{e\gamma B}}{\partial \phi_e}}{\hat{L} \frac{\partial \hat{\rho}_e}{\partial \phi_e} - \frac{\partial \hat{L}}{\partial \phi_e} \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right)} \quad (3)$$

The numerator in eq. (1) can be simplified (restoring the symmetry with the second term in the denominator) because some terms in $\hat{L} \kappa_1$ are the opposite of the corresponding ones in $\left(\hat{\rho}_{e\gamma B} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right) \kappa_2$

$$\frac{d\phi_e}{dz} = \frac{1}{z} \frac{\hat{L} \kappa'_1 + \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right) \kappa_2}{\hat{L} \frac{\partial \hat{\rho}_e}{\partial \phi_e} - \frac{\partial \hat{L}}{\partial \phi_e} \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right)} \quad (4)$$

where

$$\kappa'_1 = 4(\hat{\rho}_e + \hat{\rho}_\gamma) + \frac{3}{2} \hat{p}_B - z \frac{\partial \hat{\rho}_e}{\partial z} - z \frac{\partial \hat{\rho}_\gamma}{\partial z} - \frac{z^2 \hat{L}}{\sum_j Z_j X_j} \sum_i \left(\Delta \widehat{M}_i + \frac{3}{2z} \right) \widetilde{\Gamma}_i \quad (5)$$

The same applies to the numerator in eq. (3)

$$\frac{dX_i}{dz} = - \frac{\widehat{\Gamma}_i}{3z \widehat{H}} \frac{\kappa'_1 \frac{\partial \hat{L}}{\partial \phi_e} + \kappa_2 \frac{\partial \hat{\rho}_e}{\partial \phi_e}}{\hat{L} \frac{\partial \hat{\rho}_e}{\partial \phi_e} - \frac{\partial \hat{L}}{\partial \phi_e} \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right)} \quad (6)$$

As a consequence, eq. (A.21) can be written

$$\frac{dt}{dz} = - \frac{1}{3zH} \frac{\kappa'_1 \frac{\partial \hat{L}}{\partial \phi_e} + \kappa_2 \frac{\partial \hat{\rho}_e}{\partial \phi_e}}{\hat{L} \frac{\partial \hat{\rho}_e}{\partial \phi_e} - \frac{\partial \hat{L}}{\partial \phi_e} \left(\hat{\rho}_{e\gamma} + \hat{p}_{e\gamma B} + \frac{N(z)}{3} \right)} \quad (7)$$